Appendix 2:
HOW MANY DIFFERENT SUDOKU MATRICES ARE THERE?

After solving a number of Sudoku puzzles, you begin to wonder just how many different Sudokus there are. Will we run out of unique puzzles pretty soon? Brian Hayes has published a fascinating article in the Jan/Feb 2006 issue of the *American Scientist* entitled "Unwed Numbers". (You can find his paper on the internet by searching Google for: *Brian Hayes Unwed Numbers*.) He calculates that:

(a) **The total number of random arrangements of 9 digits in 81 cells is:**

9^81 = 1.96 x 10^77

(9^81 means 9 to the 81st power. As other examples, 4^2 = 16, 3^4 = 81, 10^6 = 1,000,000 , and 1000^2 = 1,000,000)

(b) Latin squares are arrangements of 9 digits in a 9x9 matrix, such that each row and column has one each of all 9 digits. **The total number of possible Latin squares is:**

5.525 x 10^27

(c) Sudoku matrices have the additional requirement that each of the nine 3x3 boxes also contains all 9 digits. **The total number of Sudoku matrices is:**

6.67 x 10^21

(d) But this is overcounting, since as we saw in Appendix 1, substituting a 5 for every 3 and a 1 for every 9, for example, really doesn't create a different Sudoku puzzle. Neither does rotating the matrix by 90° or reflecting it right for left change anything. If all of these symmetry operations and label permutations are taken into account, then **the total number of genuinely different Sudoku solutions is:**

5,472,730,538 = 5.5 billion

(Hayes has called my attention to the fact that the number given in his article is in error because of a confusion in defining symmetry operations. The correct figure shown above is given by Ed Russell and Frazer Jarvis at their internet site: **www.afjarvis.staff.shef.ac.uk/sudoku**.) Keep in mind that this is the total number of completed Sudoku matrices, or of Sudoku answers. Since different sets of starting digits can lead to the same answer, the total number of initial digit sets that lead to an acceptable Sudoku answer is even more mind-bogglingly large.

These numbers lead to some interesting comparisons:

(e) What fraction of the total number of arrangements of digits in a 9x9 matrix satisfies the Latin square requirements?

(1.96 x 10^77)/(5.525 x 10^27)= 3.55 x 10^49 = 35,500 x 10^45

Hence only one out of every 35 thousand billion billion billion billion billion possible arrangements of digits in a 9x9 matrix is a Latin square!

(f) Of these Latin squares, what fraction also satisfy the added Sudoku 3x3 box conditions?

(5.525 x 10^27)/(6.67 x 10^21) = 0.83 x 10^6

That is, roughly one Latin square in a million is also a valid Sudoku matrix.

(g) If you allow for permutation of the nine digit names, and for symmetry operations on the matrix, what fraction of these Sudoku-satisfying matrices are truly different?

(6.67 x 10^21)/(5.5 x 10^9) = 1.21 x 10^12

Hence only one puzzle in every 1.2 trillion possible Sudokus is genuinely unique.

In sum there are 5.5 billion different Sudokus. What does this mind-boggling number mean? Let us say that you can solve one Sudoku puzzle every 15 minutes, and can keep on solving them one after the other without food, sleep, or bathroom breaks. Then you could solve 4 x 24 x 365 = 35,040 Sudokus per year. And:

(5.5 x 10^9)/(3.5 x 10^4) = 157,000

That is, at the specified rate of four Sudokus solved per hour nonstop, to get through all possible Sudokus would require roughly 157 thousand years. If, instead of migrating out of Africa, the first *Homo sapiens sapiens* had sat down and begun solving Sudoku puzzles at this rate, he would finish solving all possible Sudokus just about now.

Another way of looking at it: I have a book of 250 Sudoku puzzles that is 2 cm thick. How many such books would be needed to print every possible Sudoku?

(5.5 x 10^9)/250 = 22 x 10^6

22 million books would be required in all. At 2 cm per book this comes to 44 million cm or 440 km of bookshelves. A single bookshelf would extend from Paris to Amsterdam, or from Boston to Philadelphia.

So—don't worry about running into the same puzzle more than once!