

10. A FIELD TEST OF PARALLEL ANALYSIS

How well does the parallel analysis method work in practice? I tested this during the last half of 2007 and beginning of 2008 by keeping track of solutions of Sudokus published in our two local newspapers: the Los Angeles Times (LAT) and the Pasadena Star News (PSN). More recently, twenty London Telegraph (TEL) Diabolical puzzles for Jan/May 2007 were added, these being available on the Mepham Group web site at www.sudoku.org.uk. Both the LAT and TEL puzzles are products of the Mepham Group, although I encountered no overlap among the puzzles tabulated here. PSN Sudokus are from Knight Features/Distributed by Universal Press syndicate, but the creator of these puzzles is not specified. LAT and TEL puzzles use four gradations: Gentle, Moderate, Tough and Diabolical, whereas PSN puzzles are ranked from one to six stars. Only Tough/Diabolical and 5/6 star puzzles were included in these tests, since the simpler ones can always be solved by the three basic methods without parallel analysis, and hence are less interesting.

The results for 100 Sudokus are shown below. Roughly equal numbers of each class of puzzle were tested.

pal:	0	1	2	3	4	No.	Sum	Avg
LAT--Tou	8	10	1	-	-	19	12	0.63
PSN--5	8	7	3	-	-	18	13	0.72
PSN--6	2	17	3	-	-	22	23	1.05
LAT--Dia	3	11	7	-	-	21	25	1.19
TEL--Dia	1	10	5	4	-	20	32	1.60
Total:	22	55	19	4	-	100	105	1.05

The pal values at the heads of columns are the number of parallel analysis levels needed to produce the correct answer. A solution that uses only the three basic methods and does not need parallel analysis is assigned pal=0. The table above tells us for example that, of the twenty-one Diabolic LAT puzzles, three did not require parallel analysis at all, eleven gave the right answer after a single level of analysis, and seven others required two levels. The only ones to demand three levels of analysis were four of the Diabolic TEL puzzles.

The mean pal value is a measure of difficulty of the puzzle categories. To calculate this mean pal value, multiply each individual pal value by its number of examples, and compute the sum. For LAT--Diabolical, for example,

$$\text{Sum} = 0 \times 3 + 1 \times 11 + 2 \times 7 = 25$$

The 21 puzzles at this level required a total of 25 levels of parallel analysis, for an average of:

$$\text{Sum/No.} = 25/21 = 1.19 \text{ levels per puzzle}$$

At the other end of the spectrum, the 19 LAT--Tough puzzles required a sum of 12 parallel analysis levels:

$$\text{Sum} = 0 \times 8 + 1 \times 10 + 2 \times 1 = 12$$

for an average per puzzle of:

$$\text{Sum/No.} = 12/19 = 0.63 \text{ levels per puzzle}$$

These two pal values, 0.63 for Tough and 1.19 for Diabolical, give you a quantitative measure of the relative difficulty of the two categories. $1.19/0.63 = 1.9$, so one can say that by this criterion, Diabolical LAT puzzles on average are twice as hard as Tough ones. The difference between levels is smaller for PSN puzzles: those with 6 stars are on average 1.45 times or half again as hard as 5-stars.

Both LAT and TEL puzzles are products of the Mepham Group in England. But the TEL puzzles in the London Telegraph seem to be consistently more difficult than the Los Angeles Times LAT set, with mean pal=1.60 rather than 1.19. Perhaps the Brits feel like making things a little bit easier for the natives in the colonies.

Pal numbers in some cases are upper bounds. If your initial solution has a high pal value, with diligent searching you may find a more direct path to the answer with a smaller pal value. As an example, one of the LAT--Dia pal=2 cases in the table above initially had pal=3, until a different way through the maze was found that required one fewer analysis steps. But pal=2 to pal=1 changes are rare, and pal=1 to pal=0 changes indicate only that you had failed to use the basic methods fully.

One more test is worth citing. Andrew Stuart has published what probably is the best available book on the classical approach to Sudokus: "The Logic of Sudoku" (Michael Mepham, 2007). His treatise lists more than thirty special methods, with names like X-wing, Sword-fish, Jellyfish, Multi-coloring, XY-chains, Forcing chains, Remote pairs, Alternating inference chains (AIC), Almost locked sets, Finned and filleted X-wings, Sashimi finned X-wings, Aligned pair exclusion (APE), Sue-de-coq, Death blossom, Guardian/broken wings, and Bi-value universal grave or BUG. Some of these are elegant and interesting, but none are absolutely necessary.

Stuart's book includes 34 sample Sudoku puzzles designed to illustrate each of these special methods. But--what fraction of these demonstration puzzles can also be solved by the three basic strategies, followed if necessary by parallel analysis? The answer is: All of them. The table below shows how many levels of parallel analysis (pal) were required to solve the same 34 puzzles by the methods that we have been describing. The three basic strategies (pal=0) sufficed for puzzles 1, 2 and 32. Other puzzles required one or two levels of parallel analysis, and seven puzzles needed three

levels, like Alternative 3 in my Plate 6.11. Puzzle 6 was especially recalcitrant and demanded four levels of analysis before cracking. It is possible that with a little work and study, one could find ways of solving one or more of the $pal=3$ or 4 cases with fewer levels.

Stuart Puzzle	pal	Stuart Puzzle	pal	Stuart Puzzle	pal	Stuart Puzzle	pal
1	0	10	3	19	1	28	1
2	0	11	1	20	3	29	2
3	1	12	1	21	2	30	3
4	2	13	3	22	1	31	2
5	3	14	2	23	1	32	0
6	4	15	2	24	2	33	1
7	1	16	3	25	2	34	3
8	1	17	1	26	1		
9	2	18	1	27	1		

There are those who object to strategies like Ariadne's thread and parallel analysis on the grounds that they are "only guesswork". I beg to differ; parallel analysis in particular employs a careful process of logical analysis. If parallel analysis is "guesswork", then all scientific research (which is what I do for a living) is also "just guesswork". In parallel analysis you frame an initial hypothesis, test it rigorously, and then frame a new hypothesis using the information obtained from the earlier trial. Knowledge accumulates from one step to the next, and is put to good use. It is the underlined portion of the sentence above that differentiates parallel analysis from Ariadne's Thread, and keeps it from being some kind of random guesswork. Parallel analysis is so powerful in helping one to solve puzzles and even to understand how to create new ones, that I regard it as much more stimulating than looking for wings, sea creatures and bugs. I haven't yet found a published Sudoku puzzle that couldn't be solved with the above four strategies plus a lot of thought. But who knows what evil lurks in the heart of a Sudokuist? (Or is that: "Sudokist"?) I would enjoy hearing about your experiences with these strategies.

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