5. LEVEL: DIABOLICAL

The methods described so far will be referred to collectively as the "three basic strategies". These will get you a long way, but they still usually fail with puzzles that are rated *Extremely Difficult* or *Diabolical*. The basic strategies alone can halt at a stalled solution, dead in the water, with no suggestion as to what to do or where to go next. A new and more powerful procedure is needed. This is the method of parallel analysis, and it will turn out to be the strongest weapon in the Sudoku-solver's arsenal. I have encountered no published Sudoku puzzle yet that could not be solved by the parallel analysis method. All that is needed is pen or pencil and a photocopier machine. But here is where Bill Chapin's matrix-drawing *Excel* file comes into its own; it is easy and rapid to use, makes the photocopier superfluous, and takes all the pain out of the most complicated of parallel solutions.

To illustrate the parallel analysis method, the initial Plate 5.1 (which I call sheet A) contains an *Extremely Difficult* or *Diabolical* Sudoku. We now have switched over to my own extension of the Chapinator file, in which each of the 81 cells begins with a 3x3 grid of digits 1 – 9. The starting set of digits is introduced, enlarging them for convenience as in sheet A. This approach replaces the 3x3 grid of dots representing digits that have been ruled out, by a 3x3 array of digits that still remain possible for each particular cell. Instead of marking the dot that represents a forbidden digit, you now simply delete that digit from the cell. If you are not using the Chapinator it still pays to prepare a hand-drawn version of a page like Plate 5.3, so one can make as many photocopies as necessary and cross out numbers by hand. But the *Excel* version is much easier to use.

In solving Plate 5.1, as usual, first delete all digits that are disallowed by the starting digit set, to obtain Plate 5.2 (which will be termed sheet B). Then use the three basic strategies seen earlier to go as far as possible toward a solution. If the analysis goes to completion, you have won. But with *Diabolical* puzzles there usually come a time when none of the basic strategies have anything more to offer. What we will call sheet C (Plate 5.3) displays the solution carried as far as one can go using the three basic strategies. It is the end of the line for the methods presented so far. The analysis is not wrong; it simply has stalled with nowhere left to go. The challenge of this chapter is how to use parallel analysis methods to break the stall observed in sheet C.

Note that Plate 5.3 has nineteen different cells that are limited to two possible digits each. The starting point in parallel analysis is to select one such cell and try each of its two possible digits in turn in a continued analysis using the basic methods of the previous chapters. Not all of these binary cells in Plate 5.3 are genuinely independent. Cells B6 and B8 are an ambiguous pair that can hold only a 4 or a 5, so when you have specified the contents of one cell you also know the contents of the other. Other binary cells on Plate 5.3 are linked as triplets or even quadruplets:

Single cells (7):	A9	G5	H6	H8	H9	I8	I9
Paired cells (1):	B6/B8						
Triplets (2):	A2/C2/C6			D3/D7/F7			
Quadruplets (1):	E2/E6/F5/G2						

Hence there are only eleven truly independent two-digit cells, and eleven different starting points for parallel analysis.

To illustrate the method in its simplest form, let us consider cell H9, and try its two options: 1 or 7. Setting H9=1 and proceeding with the basic strategies leads to the crash shown in Plate 5.4, with two conflicting 2's in row 5 and two 7's in row 6. But things go better with H9=7: the puzzle is solved correctly in Plate 5.5.

(A word of warning about crashed solutions. There is quite a bit of play or freedom in arriving at an <u>incorrect</u> solution. If you repeat this and the following analyses on your own, you may find that your crashed solutions are different in many details from my Plates 5.4, 5.6 or 5.8. But they will all exhibit the same kind of faults: <u>duplicate digits</u> in one or more rows, columns or boxes. In contrast the correct answer, in Plate 5.5 or 5.9, is unambiguous. There's a moral here. ***In Sudoku, as in real life, there are many wrong answers but only one right answer.)***

A trial with a chosen digit can have any one of three outcomes. It can:

(**@**) lead to the correct answer,

(**x**) crash with duplicate digits in one row, column or box, or simply

(s) stall once more without reaching any conclusions.

If we call the two digits in a chosen test cell Y and Z, then the possible outcomes for its tests can be:

Y	Z	Conclusions
@	**x**	The best outcome of all; digit Y is right and Z is wrong. Puzzle solved.
@	**s**	Digit Y is right but Z is ambivalent. There may be another solution lurking somewhere (but almost never in a published Sudoku).
s	**x**	You at least know that Y is right, even though the puzzle isn't yet solved.
s	**s**	Nothing new has been learned about either digit choice.
x	**x**	You have made a mistake somewhere. Either Y or Z <u>must</u> be right.
@	**@**	Two independent correct solutions: one with Y and one with Z. You won't find this situation in a published Sudoku, as designers detest such an outcome. But see the next chapter.

If either choice of digits solves the matrix the battle is over. If both choices stall you have obtained no new information. But if one choice stalls while the other one crashes, you still have learned something. <u>The digit that led to a stall is correct even though it is not sufficient by itself to solve the puzzle</u>. So add this new digit to the matrix, choose another binary cell and try again.

It might seem intuitive that the best strategy is to choose a cell that is linked to other cells, such as E2/E6/F5/G2 in this example, because choosing the digit in one of these cells also decides the digits in all the others of the set. This generally is true but not always. If you use E2=4 and its quadruplet set here, the correct answer comes up immediately. E2=7 leads to a crash. (Try this yourself.) But the D3/D7/F7 triplet is not so cooperative. A test with D3=4 leads to a crash with duplicate 1's and 4's (Plate 5.6). The other choice, D3=6, does not solve the puzzle, it only leads to another stall as seen in Plate 5.7. However, you still have gained new information: you know that D3=6 is correct even though it is not strong enough to bring about a solution. So add D3=6 to the matrix and choose another cell for testing.

In this example I added cell B6, which can hold either 4 or 5. B6=5 crashes once again, in my trial on Plate 5.8 yielding duplicate 2's and 7's in rows 6 and 5. (As mentioned earlier, if you try this you may wind up with different duplicate digits. But there will still be a crash.) In contrast B6=4 solves the puzzle (Plate 5.9). And this solution, as it must be, is identical to that seen earlier in Plate 5.5.

These two solving processes can be mapped by the tree diagrams shown in Plate 5.10. The term pal, for *parallel analysis levels*, will be used to record how many levels were needed to solve the puzzle. The trial with cell H9 required only one level of analysis to produce one solution and one crash, or pal=1. In contrast, the trial with D3 required two levels of analysis (pal=2) to yield the right answer. These two trees may seem trivial, but other especially tough Sudokus can require three or occasionally even four or more levels of analysis. In such cases a tree diagram is absolutely essential for keeping track of where you are. But in most cases the process of parallel analysis is fast, easy, and straightforward, involving only one or two levels. *Diabolical* puzzles simply are no longer diabolical if parallel analysis is used.

Parallel analysis so far has been described as studying alternative choices in cells that have two possible digits. Of course, there is nothing sacred about two digits. In Plate 5.3 you could just as well have decided to try out the 4, 6 and 7 that are possibilities in cell F3. Sometime a threefold choice is good strategy. But it carries one fundamental danger. With a twofold choice cell, [3 5], if using 3 leads to a crash and 5 to a stall, you have still learned something: the correct occupant of that cell is 5. But with a threefold cell, [3 5 6], if 3 crashes but both 5 and 6 merely stall, you haven't learned much. Either 5 or 6 must be correct, but you don't know which one. I have found that testing twofold cells is generally the better strategy, especially those that are combined with other cells into pairs, triplets or quadruplets.

In sum, parallel solution analysis is the most powerful Sudoku technique that I know of. It also can be hypnotic. Ferreting out all the different crashes and wrong answers, comparing them with one another and with the right answer, can be more interesting than merely going for the correct solution. If you continue examining solutions of Plate 5.3 on your own, you will find that it has a few more surprises to offer; surprises related to the example in the next chapter.

Plate 5.1: Initial digit set, A, with as yet uncancelled digits in other cells

Col: / Row:	A	B	C	D	E	F	G	H	I
1	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	4	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	2	6	1 2 3 4 5 6 7 8 9
2	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
3	1 2 3 4 5 6 7 8 9	1	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	3	1 2 3 4 5 6 7 8 9
4	7	1 2 3 4 5 6 7 8 9	1	9	1 2 3 4 5 6 7 8 9	2	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6
5	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
6	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	8	1 2 3 4 5 6 7 8 9	3	1	1 2 3 4 5 6 7 8 9	9
7	1 2 3 4 5 6 7 8 9	2	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	5	1 2 3 4 5 6 7 8 9
8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	7	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9
9	1 2 3 4 5 6 7 8 9	3	6	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9	9	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9

Plate 5.2: After initial crossouts of conflicting digits, B

Col: / Row:	A	B	C	D	E	F	G	H	I
1	3 5 8 9	5 7 8 9	4	1 5	1 5 7 9	1 5 7	2	6	1 5 7 8
2	2 5 9	5 6 7 9	2 5 7 9	3	1 2 4 5 7 9	8	4 5 7	1 4 7 9	1 4 5 7
3	2 5 8 9	1	2 5 7 8 9	2 4 5 6	2 4 5 7 9	4 5 6 7	4 5 7 8	3	4 5 7 8
4	7	4 5 8	1	9	4 5	2	3 4 5 8	4 8	6
5	2 3 4 5 8 9	4 5 8 9	2 3 5 8 9	1 4 5	6	1 4 5 7	3 4 5 7 8	2 4 7 8	2 3 4 5 7 8
6	6	4 5	2 5	8	4 5 7	3	1	2 4 7	9
7	1 4 8 9	2	7 8 9	1 4 6	1 3 4 8	1 4 6	3 4 6 7 8	5	1 3 4 7 8
8	1 4 5 8	4 5 8	5 8	7	1 2 3 4 5 8	9	3 4 6 8	1 2 4 8	1 2 3 4 8
9	1 4 5 8	3	6	1 2 4 5	1 2 4 5 8	1 4 5	9	1 2 4 7 8	1 2 4 7 8

Plate 5.3: Stall, C, taken as far as one can go with the basic methods

Col: / Row:	A	B	C	D	E	F	G	H	I
1	3	7	4	5	9	1	2	6	8
2	2 5	6	2 5	3	4 7	8	4 7	9	1
3	8	1	9	4 6	2	4 6 7	4 5 7	3	4 5 7
4	7	8	1	9	5	2	3	4	6
5	2 4 5	9	3	1	6	4 7	5 7	8	2 5 7
6	6	4 5	2 5	8	4 7	3	1	2 7	9
7	9	2	7	4 6	1	4 6	8	5	3
8	1 4 5	4 5	8	7	3	9	6	1 2	2 4
9	1 4	3	6	2	8	5	9	1 7	4 7

Plate 5.4: E5=6, H9=1x--crash with duplicate 2's and 7's

Col: / Row:	A	B	C	D	E	F	G	H	I
1	3	7	4	5	9	1	2	6	8
2	5	6	2	3	4	8	7	9	1
3	8	1	9	6	2	7	4	3	5
4	7	8	1	9	5	2	3	4	6
5	2	9	3	1	6	4	5	8	2
6	6	4	5	8	7	3	1	7	9
7	9	2	7	4	1	6	8	5	3
8	1	5	8	7	3	9	6	2	4
9	4	3	6	2	8	5	9	1	7

Plate 5.5: E5=6, H9=7@--successful solution

Col: / Row:	A	B	C	D	E	F	G	H	I
1	3	7	4	5	9	1	2	6	8
2	5	6	2	3	4	8	7	9	1
3	8	1	9	6	2	7	4	3	5
4	7	8	1	9	5	2	3	4	6
5	2	9	3	1	6	4	5	8	7
6	6	4	5	8	7	3	1	2	9
7	9	2	7	4	1	6	8	5	3
8	4	5	8	7	3	9	6	1	2
9	1	3	6	2	8	5	9	7	4

Plate 5.6: E5=6, D3=4x--crash with duplicate 1's and 4's

Col: / Row:	A	B	C	D	E	F	G	H	I
1	3	7	4	5	9	1	2	6	8
2	2	6	5	3	7	8	4	9	1
3	8	1	9	4	2	6	7	3	5
4	7	8	1	9	5	2	3	4	6
5	4	9	3	1	6	7	5	8	2
6	6	5	2	8	4	3	1	7	9
7	9	2	7	6	1	4	8	5	3
8	5	4	8	7	3	9	6	2	4
9	1	3	6	2	8	5	9	1	7

Plate 5.7: E5=6, D3=6s--solution stalled; but D3=6 is correct

Col: / Row:	A	B	C	D	E	F	G	H	I
1	3	7	4	5	9	1	2	6	8
2	2 5	6	2 5	3	4 7	8	4 7	9	1
3	8	1	9	6	2	4 7	4 5 7	3	4 5 7
4	7	8	1	9	5	2	3	4	6
5	2 4 5	9	3	1	6	4 7	5 7	8	2 5 7
6	6	4 5	2 5	8	4 7	3	1	2 7	9
7	9	2	7	4	1	6	8	5	3
8	1 4 5	4 5	8	7	3	9	6	1 2	2 4
9	1 4	3	6	2	8	5	9	1 7	4 7

Plate 5.8: E5=6, D3=6, B6=5x--crash with duplicate 2's and 7's

Col: / Row:	A	B	C	D	E	F	G	H	I
1	3	7	4	5	9	1	2	6	8
2	2	6	5	3	7	8	4	9	1
3	8	1	9	6	2	4	7	3	5
4	7	8	1	9	5	2	3	4	6
5	4	9	3	1	6	7	5	8	7
6	6	5	2	8	4	3	1	2	9
7	9	2	7	4	1	6	8	5	3
8	5	4	8	7	3	9	6	1	2
9	1	3	6	2	8	5	9	7	4

Plate 5.9: E5=6, D3=6, B6=4@--correct answer

Col: / Row:	A	B	C	D	E	F	G	H	I
1	3	7	4	5	9	1	2	6	8
2	5	6	2	3	4	8	7	9	1
3	8	1	9	6	2	7	4	3	5
4	7	8	1	9	5	2	3	4	6
5	2	9	3	1	6	4	5	8	7
6	6	4	5	8	7	3	1	2	9
7	9	2	7	4	1	6	8	5	3
8	4	5	8	7	3	9	6	1	2
9	1	3	6	2	8	5	9	7	4

Plate 5.10: Parallel Solutions Tree with E5=6

Starting digits, A

Initial crossouts, B

Incomplete but stalled, C

Parallel analysis, pal=1:

E5=6s

H9=1x

H9=7@

Alternative analysis, pal=2:

E5=6s

D3=4x

D3=6s

B6=5x

B6=4@

@ = correct solution

= closed ambiguous loop, two alternative solutions

s = stall, incomplete solution

x = incorrect solution with clashing digits