9. SUDOKU STRATEGIES

What makes some Sudoku puzzles harder than others? The way in which the initial set of numbers is arranged is critically important, of course. Another more general factor is the size of the starting puzzle. The more initial digits you are given to work with at the outset, the more interactions there will be, and the greater the ease of crossing out impossible locations for digits. Our *Easy* puzzle began with 36 digits, the *Medium* had 34, the *Hard* had 26 and the *Diabolical* puzzle had 25. A quick flip through a paperback of Sudoku puzzles shows that those rated *Easy* typically have around 30 to 32 starting digits, those marked *Medium* have 29 to 31, and the *Hard* and *Diabolical* puzzles have 24 to 28. These are not absolute numbers, of course; they merely express a trend that is loosely correlated with puzzle difficulty. According to Japanese designers, the smallest known number of starting digits in a published Sudoku puzzle having a unique solution is 17.

With the rise of popular enthusiasm for Sudoku puzzles, many different people and agencies have become involved in designing them. An American, Howard Garnes, first created the puzzles in 1979 by adding a 3x3 box requirement to the centuries-old Latin Square puzzle. Garnes called them "Number Place", and published them through Dell Magazines. But the real explosion of interest occurred in 1984, when Nikoli Publishing in Japan began designing their own and calling them "Sudoku", an abbreviation of a longer Japanese phrase meaning "The digits must occur only once". Sudoku came to Britain in late 2004 via the *London Times*, and to America the following spring. Today almost any American bookstore or newsstand offers an array of Sudoku books. The *Guardian* of London reprints the Nikoli puzzles. The *Times* of London and the *New York Post* feature puzzles by Wayne Gould. Will Shortz, puzzle editor for the *New York Times*, has many good paperback collections of Sudokus in print. Another designer of crosswords and other puzzles as well as Sudokus, Michael Mepham, worked with the *Daily Telegraph* in Great Britain, which provided his puzzles to many newspapers across the world, including among others the *Los Angeles Times,* the *Chicago Tribune*, the *National Post*, and *Life* magazine.

Being a Los Angeleno, I have followed Mepham's Sudokus in particular. He was a master of the *Diabolical*. Regrettably, he died in December 2006 at age 62, but his daughter Kate is carrying on in his place. The *Amazon.com* web site lists 39 paperback Sudoku collections by Mepham's firm. Most of these, and indeed almost all Sudoku books in print, contain puzzles with a broad range of difficulty from Easy to Diabolical. The Mepham Group also has four especially handy little paperbacks entitled *Sudoku to Go,* rated separately as *Gentle*, *Moderate*, *Tough* and *Diabolical*.

But this is not all. A quick visit to a Pasadena bookstore yielded fourteen other puzzle experts or publishers who have issued one or more books on Sudoku. The field is wide open; look around and take your pick.

Sudoku is a process of systematically eliminating the impossible, leading at the end (hopefully) to that which is possible. As the great detective Sherlock Holmes said on more than one occasion: *"When you have eliminated all that is impossible, then whatever remains, however improbable, must be the truth".* Holmes would have been a great Sudoku player.

Four strategies have been presented here:

1. Searching for cells that can host only one digit.
2. Finding ambiguous pairs or triples that use up two or three digits, thereby denying these digits to other neighboring cells.
3. Finding rows, columns or boxes which offer only one location for a particular digit.
4. Running systematic parallel solutions.

1. The search for single digit cells, and the generation of more such cells as numbers are added, is frequently good enough for *Easy* Sudoku puzzles.

2. For most *Medium* puzzles, one should look for ambiguous digit pairs which can be used to delete possible digits elsewhere in a row, a column or a box. With a cell containing two possible digits, you need two identical examples in order to cancel those digits from the other seven cells. Threefold and fourfold ambiguities also exist and are surprisingly common.

3. With puzzles labeled *Hard*, the above techniques eventually lead to an impasse. In such cases you need to look systematically at individual rows, columns and boxes, asking "Which digits are still missing?" and then "Which of these missing digits can fit in only one location?" The most thorough approach is to go through all nine rows, one after the other, then all nine columns, and then all nine boxes. At the end of this process, begin it again, since cancellations from new numbers that you located during the first round can make other numbers obvious.

4. If all the foregoing tactics fail, one can turn to the fourth strategy, parallel analysis. Select a cell that has only two possibilities, such as 3 and 7. You have a 50:50 chance of randomly choosing the right one. So make two xerox copies of the matrix, place a 3 in one and a 7 in the other, and continue solving the two puzzles in parallel. This will lead to a winner unless one or both of the attempts stalls. If it is the incorrect choice that stalls while the correct choice goes to completion, you have won. But if the correct choice stalls (which can happen) but the incorrect choice crashes with duplicate numbers, you know which the correct digit was, but can't use it alone to solve the puzzle. In that case, add the correct digit to the matrix, choose a different two-digit test cell and try again. It is even more powerful to pick a twofold ambiguity, if one exists. A binary ambiguity such as A4/C6=[1 8] guarantees that you will begin with either two correct digits or two incorrect ones. Having more correct digits at the start makes a stall less likely. So carry out two parallel solutions: in this example, one with A4=1 and C6=8, the other with A4=8 and C6=1. Hopefully, one of these will crash with duplicate numbers whereas the other will give the right answer. If not, then choose a different ambiguous pair, or even simply a cell having only two possibilities, and try again.

Strategies 1 - 3 frequently are best applied in reverse order. For example, if you have a good supply of single-digit cells in an *Easy* puzzle, you don't need strategy #2. But if you don't, then applying #2 is a good way of producing single-digit cells. And if you don't have many double or triple ambiguities that permit cancellations, then a systematic application of strategy #3 is a good way to generate them. A good plan of attack is first to fill in as many single digits as you can find, canceling their mates in row, column and box. Then use strategy #3 to cancel as many choices as you can, and follow this by searching through the remaining permitted digits for double or triple ambiguities.